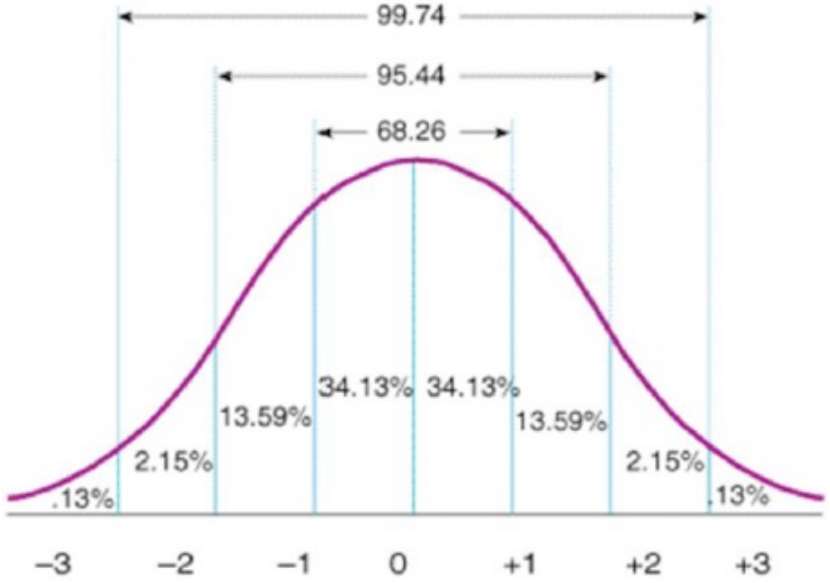
SM3 HW 12.4 Z-Scores and the Standard Normal Distribution

OBJECTIVES:

- Find the z-score for the value of any normally distributed variable
- Understand the meaning of a z-score as a measure of position within a distribution.
- Use technology or the standard normal table to find the probability of events.
- Use z-scores to compare values from two different normal distributions.

VOCABULARY:

- **Standard Normal Distribution** converts all values in the normal distribution to a standardized score called a z-score. The standard normal distribution has a mean=0 ($\mu = 0$) and standard deviation=1 ($\sigma = 1$)



- **Z-Score** is a standardized value of any normally distributed value. The sign of the z-score indicates its location relative to the mean, negative z-scores lie to the left of the mean and positive z-scores are to the right of the mean. A z-score tells you how many standard deviations away from the mean a given value lies in its normal distribution. The formula for finding a z-score is:

$$z = \frac{value - mean}{standard\ deviation} = \frac{x - \mu}{\sigma}$$

  Z-scores have no units, the farther the z-score is from 0 tells you it is closer to the tails of the distribution. Converting a normally distributed value to a z-score allows you to compare it to other values from different normal distributions.
- The **Standard Normal Table** sometimes called "Table A" gives the probabilities of getting a z-score at or below the z-score for your variable. It can be used to find the probability of events for any z-score.

Example 1: Joey scored a 83 on his Chemistry exam which is normally distributed with a mean $\mu = 71$ and a standard deviation $\sigma = 7$. Sabrina scored a 78 on her Statistics exam which is also normally distributed with a mean $\mu = 73$ and a standard deviation $\sigma = 2$

1. What is the z-score for Joey's test?

$$z = \frac{x - \mu}{\sigma} = \frac{83 - 71}{7} = 1.71$$

2. What is the z-score for Sabrina's test?

$$z = \frac{x - \mu}{\sigma} = \frac{78 - 73}{2} = 2.50$$

3. Which student scored best relative to the other students?
Sabrina scored better relative to the other students because her z-score was the higher of the two

4. What is the probability of scoring as high or higher on the Chemistry test as Joey did?
Using Table A: *the probability of getting a LOWER score from table A is .9564, we want the probability of getting a HIGHER score so we give the complement of .9564 which is 1-.9564 =* ***.0436***
Using the calculator: *probability for the standard normal curve is the same for any other normal distribution with a mean of 0 and a standard deviation of 1 so we use Normalcdf:*

Calculator:

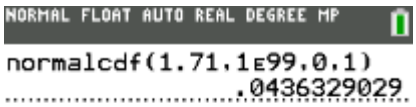


Table A:
z=1.71 and we want to find the > (right end probability) so we can switch the sign to -1.71 and use the table value (example on left) or subtract the table value for the left end probability from 1 (example on the right)

Greater than probability so I switched the sign

z	.00	.01	.02
−1.8	.0359	.0351	.0344
−1.7	.0446	.0436	.0427
−1.6	.0548	.0537	.0526
−1.5	.0668	.0655	.0643
−1.4	.0808	.0793	.0778

z	.00	.01	.02
1.6	.9452	.9463	.9474
1.7	.9554	.9564	.9573
1.8	.9641	.9649	.9656
1.9	.9713	.9719	.9726

$1 - .9564 = .0436$

5. What is the probability of scoring as high or higher on the Statistics test as Sabrina did?
Same process as #4

Calculator:

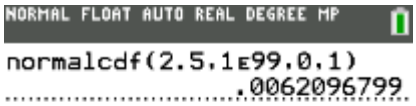


Table A: (sign was switched for > probability)

z	.00	.01	.02
−2.6	.0047	.0045	.0044
−2.5	.0062	.0060	.0059
−2.4	.0082	.0080	.0078

.0062

6. What percent of test takers had z-scores between Joey and Sabrina's?
This a normal calculation between z-scores of 1.71 and 2.5 :

Calculator:

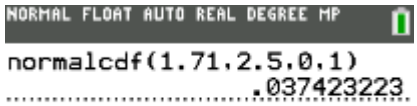
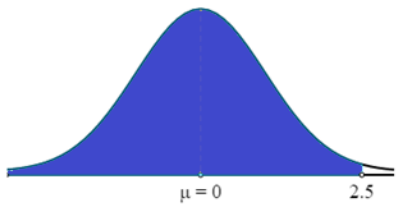
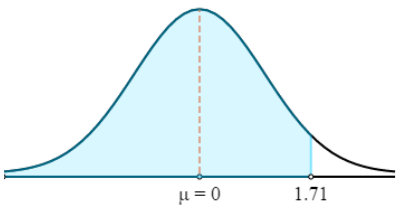


Table A: We have to find the probability for < 2.5 and the probability for <1.71 and then subtract the two.

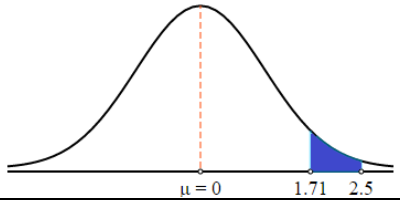
<2.5:



<1.71



<2.5 - <1.71:



12.4 EXERCISES **NAME_____**

You have to make a final decision on two applicants. They are both wonderful students with the very same G.P.A. and class rankings. It all comes down to their test scores. Student A took the ACT and received a score of 29 in mathematics. Student B took the SAT and received a score of 680 in mathematics. Since you are an expert in college entrance exams, you know that both tests are designed to be normally distributed. A perfect ACT is 36. The ACT mathematics section has a mean of 21 and standard deviation of 5.3. A perfect score on the SAT math section is 800. The SAT mathematics section has a mean of 516 and a standard deviation of 116.

1. What was Student A's z-score?

2. What was Student B's z-score?

3. Which student would you choose? Why?

4. What is the probability that a student scored better than Student A on the ACT test?

5. What proportion of students would have z-scores that fell between Student A and Student B?

Let's say you had two imaginary test takers, Jack and Jill. Jack's z-score was 1.49 and Jill's z-score was 0.89.

6. What percent of the test takers scored below Jack? What percent scored above Jack?

7. What percent of the test takers scored below Jill? What percent scored above Jill?

8. What percent of the test takers had scores between Jack's and Jill's scores?

9. Jack and Jill's friend, Jason, scored -1.49. Find the number of test takers that scored above him without using a table or technology. Explain your strategy.

At South Beach High School, there are 2500 students attending. Mariana surveys 40 of her friends where they prefer to eat lunch. She created the following two---way table showing her results:

	9th Grade	10th Grade	11th Grade	12th Grade	Totals
School Cafeteria	18	6	2	1	**28**
Off Campus	2	4	3	4	**12**
Totals	**20**	**10**	**5**	**5**	**40**

Mariana plans to use her data to answer the following questions:

I. Do students prefer to eat on campus or off campus overall?

II. Is there a difference between grade levels for where students prefer to eat lunch?

10. In Mariana's sample, what percent of students prefer school lunch? What percent prefer to eat off campus?

11. For each grade level in her sample, determine the percent of students that prefer school lunch and the percent that prefer off campus lunch. Do you notice anything unusual?

12. Based on her sample, Mariana concludes that students at South Beach High School overall prefer school lunch. Do you agree or disagree? Why?

A company makes a mean monthly income of $20,300 with a standard deviation of $3,200. In one given month the company makes $29,500.

13. Find the z-score.

14. Assuming the companies monthly income is Normal, what percent of the time does the company make more than this amount? Less than?

15. What percent of the time does the company make between $15,000 and $25,000?

16. If the company needs to make $16,400 in order to break even, how likely in a given month is the company to make a profit?

On the Wechsler Adult Intelligence Scale, an average IQ is 100 with a standard deviation of 15 units.

17. IQ scores between 90 and 109 are considered average. Assuming IQ scores follow a Normal distribution, what percent of people are considered average?

18. One measure of Genius is an IQ score of above 135. What percent of people are considered genius?

19. Einstein had an IQ score of 160. What is his z---score?

20. What is the probability of an individual having a higher IQ than Einstein?